Lecture-1

Faraday’s Law, transformer and motional electromotive forces,

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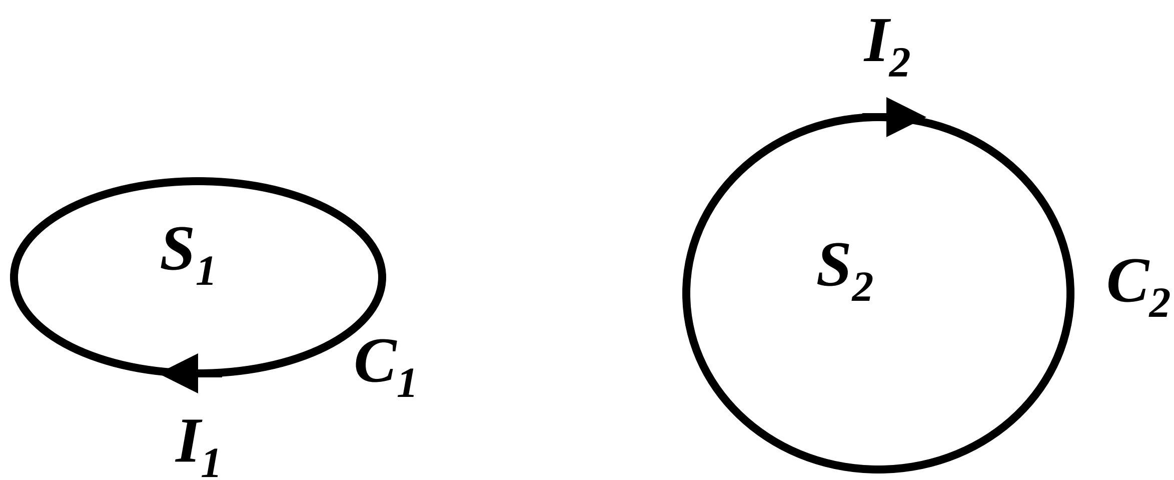
Flux Linkage

- Consider two magnetically coupled circuits



Flux Linkage (Cont'd)

- The magnetic flux produced I_1 linking the surface S_2 is given by

$$\Psi_{12} = \int_{S_2} \underline{B}_1 \cdot d\,\underline{s}_2$$

- If the circuit C_2 comprises N_2 turns and the circuit C_1 comprises N_1 turns, then the total flux linkage is given by

$$\Lambda_{12} = N_1 N_2 \Psi_{12} = N_1 N_2 \int_{S_2} \underline{B}_1 \cdot d\,\underline{s}_2$$

Mutual Inductance

- The *mutual inductance* between two circuits is the magnetic flux linkage to one circuit per unit current in the other circuit:

$$\boxed{L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_1 N_2 \Psi_{12}}{I_1}}$$

Neumann Formula for Mutual Inductance

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_1 N_2 \Psi_{12}}{I_1} = \frac{N_1 N_2}{I_1} \int_{S_2} \underline{B}_1 \cdot d\underline{s}_2$$

$$= \frac{N_1 N_2}{I_1} \oint_{C_2} \underline{A}_1 \cdot d\underline{l}_2$$

Neumann Formula for Mutual Inductance (Cont'd)

$$L_{12} = \frac{N_1 N_2}{I_1} \oint_{C_2} \underline{A}_1 \cdot d\underline{l}_2$$

$$= \frac{\mu_0 N_1 N_2}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\underline{l}_1 \cdot d\underline{l}_2}{R_{12}}$$

$$\underline{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint_{C_1} \frac{d\underline{l}_1}{R_{12}}$$

Neumann Formula for Mutual Inductance (Cont'd)

- The Neumann formula for mutual inductance tells us that
  - $L_{12} = L_{21}$
  - the mutual inductance depends only on the geometry of the conductors and not on the current

Self Inductance

- ***Self inductance*** is a special case of mutual inductance.
- The *self inductance* of a circuit is the ratio of the self magnetic flux linkage to the current producing it:

$$\boxed{L_{11} = \frac{\Lambda_{11}}{I_1} = \frac{N_1^{\ 2}\,\Psi_{11}}{I_1}}$$

Self Inductance (Cont'd)

- For an isolated circuit, we call the self inductance, ***inductance***, and evaluate it using

$$\boxed{L = \frac{\Lambda}{I} = \frac{N^2\Psi}{I}}$$

Self-Inductance

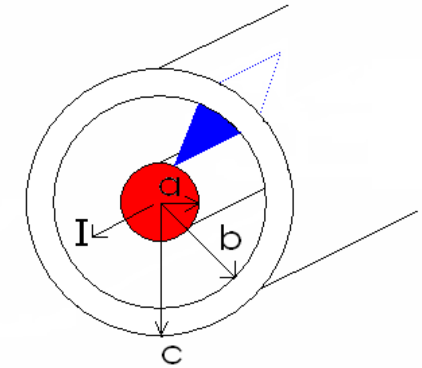
- Formula by Definition

$$L \equiv \frac{N\Phi_m}{I} = \frac{\text{flux linkage}}{\text{current through each turn}}$$

  - Applies to linear magnetic materials only
  - Units:

$$L\,[\text{Henry}] = [\text{H}] = [\text{Wb/A}] = [\text{T}\cdot\text{m}^2/\text{A}]$$

Inductance of Coaxial Cable



- Magnetic Flux

$$\Phi_m = \iint_S \vec{B} \cdot d\vec{S} = \iint_S \left(\frac{\mu I}{2\pi\rho}\hat{\phi}\right) \cdot (\hat{\phi} d\rho dz)$$

$$= \int_a^b \int_0^d \frac{\mu I}{2\pi\rho} d\rho dz = \frac{\mu I d}{2\pi} \ln(b/a)$$

- Inductance

$$L \equiv \frac{\Phi_m}{I} = \frac{\mu d}{2\pi} \ln(b/a) \ \ [\mathrm{H}]$$

$$\text{or} \quad \frac{L}{d} = \frac{\mu}{2\pi} \ln(b/a) \ [\mathrm{H/m}]$$

(as commonly used in transmission line theory)

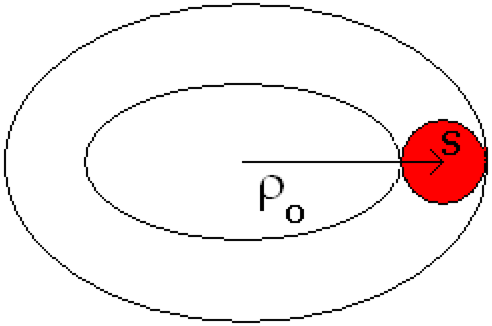
Inductance of Toroid

- Magnetic Flux Density

$$B_\phi = \frac{\mu NI}{2\pi\rho} \quad [\mathrm{T}] = [\mathrm{Wb/m^2}]$$

- Magnetic Flux

$$\Phi_m = \iint_S \vec{B} \cdot d\vec{S}$$



- If core small vs. toroid

$$\Phi_m = \vec{B} \cdot \vec{S}$$

$$= \frac{\mu NIS}{2\pi\rho_0} \quad (\text{if } S << \rho_0{}^2)$$

$$\text{where S} \equiv \text{cross section area of the toroid core}$$

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Inductance of Toroid

- Inductance $$L \equiv \frac{N\Phi_m}{I} = \frac{\mu N^2 S}{2\pi\rho_0} \quad [\mathrm{H}]$$
  – Result assumes that no flux escapes through gaps in the windings (actual L may be less)

  – In practice, empirical formulas are often used to adjust the basic formula for factors such as winding (density) and pitch (angle) of the wiring around the core

Alternative Approaches

- Self-inductance in terms of
  - Energy $$W_H = \frac{1}{2} L I^2 \;\Rightarrow\; L = \frac{2W_H}{I^2}$$
  - Vector magnetic potential (**A**)
  - Estimate by Curvilinear Square Field Map method

Energy Stored in Magnetic Field

- The magnetic energy stored in a region permeated by a magnetic field is given by

$$\boxed{W_m = \frac{1}{2}\int_V \underline{B}\cdot\underline{H}\,dv = \frac{1}{2}\int_V \mu H^2\,dv}$$

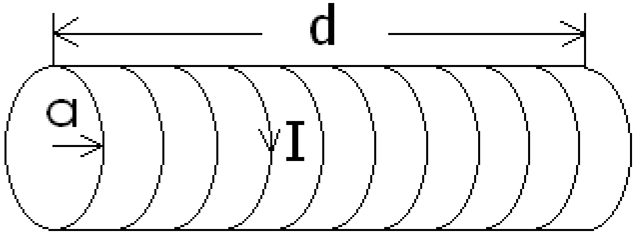
Energy Stored in an Inductor

- The magnetic energy stored in an inductor is given by

$$\boxed{W_m = \frac{1}{2} L I^2}$$

Inductance of a Long Straight Solenoid

- Energy Approach



$$W_H = \frac{1}{2}\iiint_{vol.} \vec{B}\cdot\vec{H}dv = \frac{\mu}{2}\iiint_{vol.} H^2 dv$$

$$\text{where } H = \frac{NI}{d} \quad \text{for this solenoid}$$

$$\therefore W_H = \frac{\mu}{2}\iiint_{vol.} \frac{N^2 I^2}{d^2} dv = \frac{\mu N^2 I^2}{2d^2} \iint_{S(core)} dS \int_0^d dz$$

$$\Rightarrow W_H = \frac{\mu N^2 I^2 S}{2d} \quad \text{where } S = \pi a^2 \text{ for a circular core}$$

- Inductance $$L = \frac{2W_H}{I^2} = \frac{\mu N^2 S}{d}$$

Internal Inductance of a Long Straight Wire

- Significance: an especially important issue for HF circuits since

$$|Z_L| \cong X_L = \omega L \quad \therefore \quad \omega \uparrow \Rightarrow |Z_L|$$

- Energy approach (for wire of radius a)

$$W_H = \frac{1}{2} \iiint_{vol.} \vec{B} \cdot \vec{H} dv = \frac{\mu}{2} \iiint_{vol.} \left(\frac{I\rho}{2\pi a^2}\right)^2 \rho d\rho d\phi dz$$

$$= \frac{\mu I^2}{8\pi^2 a^4} \int_0^a \rho^3 \int_0^{2\pi} d\phi \int_0^l dz$$

$$= \frac{\mu I^2}{8\pi^2 a^4} (a^4/4)(2\pi)(l) = \frac{\mu I^2 l}{16\pi}$$

Internal Inductance of a Long Straight Wire

- Expressing Inductance in terms of energy

$$L = \frac{2W_H}{I^2} = \frac{2\left(\frac{\mu I^2 l}{16\pi}\right)}{I^2} = \frac{\mu l}{8\pi}$$

$$\text{or} \quad \frac{L}{l} = \frac{\mu}{8\pi}$$

- Note: this result for a straight piece of wire implies an important rule of thumb for HF discrete component circuit design:

  "keep all lead lengths as short as possible"

Example of Calculating Self-Inductance

- Exercise 1

  Find: the self-inductance of

  a) a 3.5 m length of coax cable with a = 0.8 mm and b = 4 mm, filled with a material for which

  $\mu_r = 50$.

$$L = \frac{\mu d}{2\pi}\ln(b/a) = \frac{\mu_r \mu_0 d}{2\pi}\ln(b/a)$$

$$= \frac{(50)(4\pi \times 10^{-7}\,\mathrm{H/m})(3.5\mathrm{m})}{2\pi}\ln\left(\frac{4}{0.8}\right)$$

$$L = 56.3\ \mu\mathrm{H}$$

Example of Calculating Self-Inductance

- Exercise 1 (continued)

  Find: the self-inductance of

  b) a solenoid having a length of 50 cm and 500 turns about a cylindrical core of 2.0 cm radius in which μ_r = 50 for 0 < ρ < 0.5 cm and μ_r = 1 for 0.5 < ρ < 2.0 cm

$$L = \frac{\mu N^2 S}{d} = \frac{\mu_i N^2 S_i}{d} + \frac{\mu_o N^2 S_o}{d} = (\mu_i S_i + \mu_o S_o)\frac{N^2}{d} = (50 S_i + S_o)\frac{\mu_0 N^2}{d}$$

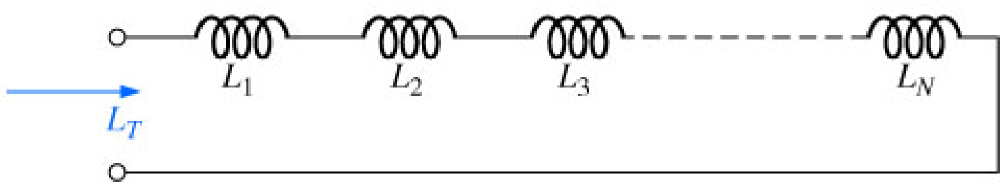
$$\text{where } S_i = \pi(.005\text{ m})^2 = 78.5\times10^{-6}\ \text{m}^2$$

$$\text{and} \quad S_o = \pi[(.020\text{ m})^2 - (.005\text{ m})^2] = 1.18\times10^{-3}\ \text{m}^2$$

$$\therefore L = [(50)(78.5\times10^{-6}) + 1.18\times10^{-3}]\frac{(4\pi\times10^{-7})(500)^2}{0.50} = 3.2\text{ mH}$$

Inductors in Series and in Parallel

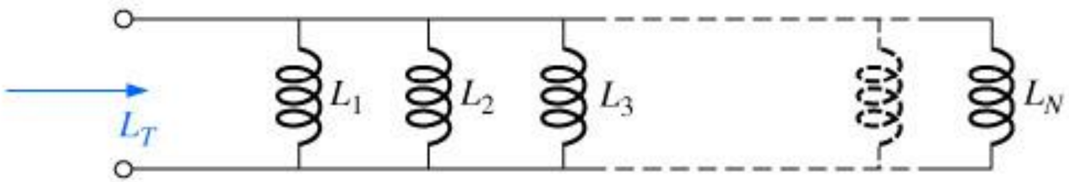
- Inductors, like resistors and capacitors, can be placed in series
  - Increasing levels of inductance can be obtained by placing inductors in series



$$\boxed{L_T = L_1 + L_2 + L_3 + \cdots + L_N}$$

Inductors in Series and in Parallel

- Inductors, like resistors and capacitors, can be placed in parallel.
  - Decreasing levels of inductance can be obtained by placing inductors in parallel.



$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

Mutual Inductance

- Significant when current in one conductor produces a flux that links through the path of a 2nd separate one and vice versa

- Defined in terms of magnetic flux (Φ_m)

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1} \equiv \text{mutual inductance between circuits 1 and 2}$$

where $\Phi_{12} \equiv$ the flux produced by I_1 that links the path of I_2

and $N_2 \equiv$ the # of turns in circuit 2

Mutual Inductance Between Circular Loops

- A circular loop of conducting wire of radius *a* carries current *I*. Find the magnetic field on the axis of the loop a distance *h* from the plane of the loop by direct integration of the Biot-Savart Law.

- If a small circular circuit of radius   is placed at this position (so that the magnetic field may be considered uniform over the area of the small loop) such that the planes of the two circuits are parallel, find the mutual inductance between them.